Properties of and Generalized Full-Wave Transmission Line Models for Hybrid (Bi)(an)isotropic Waveguides

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Abstract— **In this contribution single and coupled equivalent transmission lines are developed for the propagation of modes in reciprocal and nonreciprocal, anisotropic, bi-isotropic, and bi-anisotropic waveguides. The transmission lines are described by the generalized telegrapher's equations. In order to develop these transmission line models some properties, related to reciprocity, bi-directionality, and mirroring, of general waveguides and generalized transmission lines are investigated. The transmission line models are based on the reciprocity theorem and are valid for arbitrary frequencies.**

I. Introduction

THE representation of the propagation of modes in a waveguide by a transmission line in a circuit representation is in use for a very long time. For TEM-waveguides, such as the coaxial cable, this transmission line representation follows immediately from the Maxwell equations as is shown in every basic electromagnetics course. Later [1], it was shown that at low frequencies, i.e., in the quasi-TEM limit, the transmission line representation is still equivalent to the propagation of eigenmodes in hybrid multiconductor waveguides. These hybrid waveguides are nonhomogeneous and hence do not propagate pure TEM modes. With the application of the microstrip line at higher frequencies, where the quasi-TEM assumption becomes questionable, the need arose for transmission line models valid at arbitrary frequencies. Many different solutions for this problem were put forward in the past. We refer the reader to [2] for an extensive overview. However, all of these transmission line models were based on the assumption that the same power has to be transported in the transmission line as in the waveguide. In [3], it was shown that this causes inconsistencies for lossy waveguides. Reciprocal waveguides were represented by nonreciprocal transmission lines. In [3], new types of transmission line models were proposed based on the Lorentz reciprocity principle rather than on the power assumption. Under this assumption the conservation of reciprocity was automatically guaranteed. These transmission line models were then successfully extended to incorporate the influence of externally impinging waves on the waveguide [4] or to the interconnection of different waveguides [5]. They were in particular applied to wire transmission lines for power lines [6]. It has to be mentioned also that in [3] the concept of transmission line modeling, which was classically only used for multiconductor transmission lines, was extended to incorporate dielectric and optical waveguides.

The representation of for example a microstrip on a biased and hence nonreciprocal ferrite substrate by a classical transmission line causes problems. Indeed a classical transmission line is traditional mirroring and bi-directional [7]. This means that for each mode propagating in one direction there is a mode propagating in the other direction with the same propagation constant (bi-directional) and with the same modal voltages and currents (mirroring). Remark that our definition of bi-directionality differs from the one used in [7] where bi-directionality was reserved for mirroring waveguides. A glossary defining terms can be found in Appendix A. In the ferrite situation this is no longer the case. In order to correctly represent this nonreciprocal waveguide we need a generalized transmission line. As we will show in this paper these problems are not restricted to nonreciprocal waveguides but similar problems also occur in certain anisotropic reciprocal waveguides. Again, these waveguides need a generalized transmission line concept. Generalized transmission line concepts also come into play when there are bi-isotropic or bi-anisotropic materials involved in the waveguides. Since they are important in the optical frequency range and recently also got a lot of attention in the microwave frequency range, we will also include them in this paper. In [8] and [9] the quasi-TEM theory of [1] was generalized to multiconductor transmission lines in bi-anisotropic inhomogeneous media. It was shown that the generalized transmission line concept rigorously followed from the Maxwell equations under the quasi-TEM assumption.

In this contribution, we will extend the high-frequency transmission line models developed in [3] to generalized transmission lines. First, this will require a study of some aspects of reciprocal and nonreciprocal waveguides and generalized transmission lines. In particular, the problem of bi-directionality of reciprocal waveguides will be addressed and a proof will be given that reciprocal waveguides are bi-directional although they are not necessary mirroring waveguides. In the previous literature it was almost assumed evident that a reciprocal waveguide is bi-directional although, in the most general case, this does not follow immediately from Maxwell's equations. In tracing back literature we encoun-

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tered [10] and [11] where a proof was given based on a network concept for nonreciprocal gyrotropic and anisotropic media. In the sequel we will comment on that proof. For nonreciprocal waveguides and transmission lines the concept of adjoint waveguides and transmission lines is studied. Also in the first part of the paper the problem of the nonmirroring property of bi-directional or mutual bi-directional waveguides is investigated and a relation is constructed between the modal profiles of two oppositely propagating modes with the same propagation constant in a waveguide as well as on a transmission line.

In the second part of the paper the attention goes to the construction of the actual generalized transmission line models. First, generalized single transmission line models for reciprocal and nonreciprocal waveguides are constructed. These transmission line models are based on the Lorentz reciprocity theorem. One of the particular problems in these models is the fixation of the amplitudes of the modes in the waveguide and the transmission line. This problem has already been studied in [12] for the plane wave propagation of waves in nonreciprocal bi-anisotropic materials. Throughout this paper we will call this fixation of the amplitudes normalization. Care should however be taken with this normalization because, as in [3] and [12], we will use a reciprocity integral for this normalization which is not a norm in the mathematical sense of the word. In the last part of the paper generalized coupled transmission line models are studied. In this case, however, we will not be able to give closed form expressions for the transmission line parameters as a function of waveguide quantities. We have to restrict ourselves to implicit formulas which require numerical solutions.

In [2] and [3], different transmission line models were introduced based on the assumption that the current or the voltage in the transmission line has a physical meaning. The terms RI-model and RV-model were introduced in [3] to make the distinction between these current and voltage models, respectively. The "R" indicates that the models are not based on power but rather on reciprocity. In [3], also more general models, applicable for optical and dielectric waveguides were introduced. In this paper we will restrict ourselves to RI-models, the extension to the RV-models is straightforward.

II. Reciprocity and Bi-Directionality

If $[\mathbf{e}_a(\mathbf{r}), \mathbf{h}_a(\mathbf{r})]$ and $[\tilde{\mathbf{e}}_b(\mathbf{r}), \tilde{\mathbf{h}}_b(\mathbf{r})]$ are two solutions of the Maxwell equations in a source free volume V then they satisfy the Lorentz reciprocity theorem

$$\int_{\delta V} [\mathbf{h}_a(\mathbf{r}) \times \tilde{\mathbf{e}}_b(\mathbf{r}) - \tilde{\mathbf{h}}_b(\mathbf{r}) \times \mathbf{e}_a(\mathbf{r})].\mathbf{u}_n \, dS = 0 \tag{1}$$

where δV is the boundary surface of this volume V and where $\mathbf{u}_n$ is the external normal on this surface. This reciprocity theorem is also valid in nonreciprocal media if the solution $[\tilde{\mathbf{e}}_b(\mathbf{r}), \tilde{\mathbf{h}}_b(\mathbf{r})]$, indicated by a tilde, is a solution of the Maxwell equations in the adjoint medium in the volume V. The theorem is easily checked and is a generalization of the one presented in [13] or [14]. The adjoint medium has reciprocal material parameters compared to the original medium

$$\tilde{\bar{\bar{\epsilon}}} = \bar{\bar{\epsilon}}^T, \tag{2}$$

$$\tilde{\bar{\bar{\mu}}} = \bar{\bar{\mu}}^T, \tag{3}$$

$$\tilde{\bar{\bar{\xi}}} = -\bar{\bar{\zeta}}^T \tag{4}$$

and

$$\tilde{\bar{\bar{\zeta}}} = -\bar{\bar{\xi}}^T \tag{5}$$

where the constitutive equations for a general bi-anisotropic medium are cast in the form

$$\mathbf{d}(\mathbf{r}) = \bar{\bar{\epsilon}} \cdot \mathbf{e}(\mathbf{r}) + \bar{\bar{\xi}} \cdot \mathbf{h}(\mathbf{r}), \tag{6}$$

$$\mathbf{b}(\mathbf{r}) = \bar{\bar{\zeta}} \cdot \mathbf{e}(\mathbf{r}) + \bar{\bar{\mu}} \cdot \mathbf{h}(\mathbf{r}). \tag{7}$$

Assume that the a-field is an eigenmode propagating along the z-axis with propagation constant γ_a in a hybrid waveguide. The modal field can be written as

$$\mathbf{e}_a(\mathbf{r}) = \mathbf{E}_a(x, y) \exp(-\gamma_a z), \tag{8}$$

$$\mathbf{h}_a(\mathbf{r}) = \mathbf{H}_a(x, y) \exp(-\gamma_a z). \tag{9}$$

where $\mathbf{E}_a(x, y)$ and $\mathbf{H}_a(x, y)$ are the modal field patterns. Similarly assume that the b-field is an eigenmode propagating in the adjoint waveguide with propagation constant $\tilde{\gamma}_b$. Applying the Lorentz reciprocity theorem (1) to a section of arbitrary length L of the waveguide between $z = 0$ and $z = L$ for these modes, results in

$$\{1 - \exp[-(\gamma_a + \tilde{\gamma}_b)L]\} \int_S [\mathbf{H}_{t,a}(x, y) \times \tilde{\mathbf{E}}_{t,b}(x, y) - \tilde{\mathbf{H}}_{t,b}(x, y) \times \mathbf{E}_{t,a}(x, y)].\mathbf{u}_z \, dS = 0 \tag{10}$$

where $\mathbf{u}_z$ is the z-directed unit vector and where the subscript "t" denotes the transverse components, i.e., the components in the cross-section S of the waveguide. The relation (10) is the generalized orthogonality relation for eigenmodes in reciprocal and nonreciprocal bi-anisotropic waveguides. The relation states that when $\gamma_a \neq -\tilde{\gamma}_b$ the integral has to be zero.

It is easy to show, by reversing the z-axis in Maxwell's equations, for anisotropic waveguide that when γ_a is a propagation constant of an eigenmode that there is an eigenmode with propagation constant $-\gamma_a$ propagating in the opposite direction. In general one automatically accepts that this bi-directionality property can be generalized to arbitrary reciprocal bi-anisotropic waveguides. However, it is not possible to show this property immediately from Maxwell's equations by reversing the z-axis (see also next section). In this section we will give a short proof based on the Lorentz reciprocity theorem (10). We will prove that the original waveguide and the adjoint waveguide are mutual bi-directional. This means that if γ_a is a propagation constant that $\tilde{\gamma}_b = -\gamma_a$ is also a propagation constant in the adjoint waveguide.

First, assume a hybrid waveguide consisting of isotropic material. From Maxwell's equations it follows immediately, by reversing the z-axis, that if $[\gamma_{a,0}, \mathbf{H}_{t,a,0}(x, y), \mathbf{E}_{t,a,0}(x, y)]$ is an eigenmode that also $[\tilde{\gamma}_{b,0} = -\gamma_{a,0}, \tilde{\mathbf{H}}_{t,b,0}(x, y) = -\mathbf{H}_{t,a,0}(x, y), \tilde{\mathbf{E}}_{t,b,0}(x, y) = \mathbf{E}_{t,a,0}(x, y)]$ is an eigenmode. Inserting these equations in (10) shows of course that the factor in front of the integral vanishes and that we can choose the amplitude of the a mode such that

$$\frac{1}{2}\int_S [\mathbf{H}_{t,a,0}(x,y)\times\mathbf{E}_{t,a,0}(x,y)].\mathbf{u}_z\,dS = 1. \quad (11)$$

Now change the isotropic material parameters in such a way that they become slightly bi-anisotropic. The material parameters of the adjoint waveguide are changed accordingly. This will change the field patterns and propagation constants of both the a and b modes by a small amount. We write these modes as $[\gamma_{a,1}=\gamma_{a,0}+\delta\gamma_{a,1},\ \mathbf{H}_{t,a,1}(x,y)=\mathbf{H}_{t,a,0}(x,y)+\delta\mathbf{H}_{t,a,1}(x,y),\ \mathbf{E}_{t,a,1}(x,y)=\mathbf{E}_{t,a,0}(x,y)+\delta\mathbf{E}_{t,a,1}(x,y)]$ and $[\tilde{\gamma}_{b,1}=-\gamma_{a,0}+\delta\tilde{\gamma}_{b,1},\ \tilde{\mathbf{H}}_{t,b,1}(x,y)=-\mathbf{H}_{t,a,0}(x,y)+\delta\tilde{\mathbf{H}}_{t,b,1}(x,y),\ \tilde{\mathbf{E}}_{t,b,1}(x,y)=\mathbf{E}_{t,a,0}(x,y)+\delta\tilde{\mathbf{E}}_{t,b,1}(x,y)]$. Inserting these modes in (10) results after a few calculations in

$$\begin{aligned}&\{1-\exp[(\delta\gamma_{a,1}+\delta\tilde{\gamma}_{b,1})L]\}\\ &\times\left[4+\int_S(\delta\mathbf{H}_{t,a,1}\times\tilde{\mathbf{E}}_{t,b,0}-\tilde{\mathbf{H}}_{t,b,0}\times\delta\mathbf{E}_{t,a,1}).\mathbf{u}_z\,dS\right.\\ &+\int_S(\mathbf{H}_{t,a,0}\times\delta\tilde{\mathbf{E}}_{t,b,1}-\delta\tilde{\mathbf{H}}_{t,b,1}\times\mathbf{E}_{t,a,0}).\mathbf{u}_z\,dS\\ &\left.+\int_S(\delta\mathbf{H}_{t,a,1}\times\delta\tilde{\mathbf{E}}_{t,b,1}-\delta\tilde{\mathbf{H}}_{t,b,1}\times\delta\mathbf{E}_{t,a,1}).\mathbf{u}_z\,dS\right]=0.\end{aligned} \quad (12)$$

When the material parameters are only slightly changed, the second factor in this expression will not vanish because, except for the fixed term "4," the other terms in the second factor are at least of first order in the changes. This means that the first factor has to be zero, which in turn means that $\delta\gamma_{a,1}=-\delta\tilde{\gamma}_{b,1}$ or that $\gamma_{a,1}=-\tilde{\gamma}_{b,1}$.

In a next step we start from the "a, 1" and "b, 1" modes. First we choose the amplitudes of the modes again such that

$$\begin{aligned}\frac{1}{2}\int_S &[\mathbf{H}_{t,a,1}(x,y)\times\mathbf{E}_{t,b,1}(x,y)\\ &-\mathbf{H}_{t,b,1}(x,y)\times\mathbf{E}_{t,a,1}(x,y)].\mathbf{u}_z\,dS=1.\end{aligned} \quad (13)$$

Now we can again change the material parameters by a small amount and proceed as in (12). By repeating this process over and over one proves that a bi-anisotropic waveguide and his adjoint waveguide are mutual bi-directional or in particular that a reciprocal waveguide is bi-directional.

One could argue why not just take a finite piece of the reciprocal waveguide and consider the two ends of the guide as ports to a microwave network as suggested in [10]. Then attach to one port a transmitter and to the other port a receiver. The transmitter should convert a voltage or current in a lumped element to a single mode in the waveguide and vice versa for the receiver. In a next step both the transmitter and receiver are interchanged and then from the argument that the same voltage or current is recorded in the receiver it follows that the propagation constants are opposite according to [10]. The problem in this setup is that, as we will discuss in the next section, the modes propagating in opposite direction can have totally different modal field profiles. The used transmitter in the setup however should generate only one mode when connected to the left and to the right hand side. It may be that such a transmitter could be conceived but it is certainly not evident. If the transmitter generates more than one mode then it is also not evident to draw conclusions regarding one mode. This discussion puts at least some questions on the proof in [10]. Remark that the proof in [10] is also restricted to closed waveguides and gyrotropic anisotropic materials. Our proof has no need to introduce some network element and follows directly from Maxwell's equations via the reciprocity theorem. Since only the reciprocity theorem is involved the proof applies to all mutual reciprocal linear eigenvalue systems.

III. Relation Between the Modal Field Patterns

In an isotropic waveguide we know that the modal field patterns of two oppositely propagating modes, with the same propagation constant, are what we will call mirror images. Indeed, it can be easily checked that the transformation formulas

$$\begin{aligned}&\gamma\rightarrow-\gamma,\\ &\mathbf{E}_t\rightarrow\mathbf{E}_t,\\ &E_z\rightarrow-E_z,\\ &\mathbf{H}_t\rightarrow-\mathbf{H}_t,\\ &H_z\rightarrow H_z.\end{aligned} \quad (14)$$

transform the Maxwell curl equations into themselves. Now we wonder for which bi-anisotropic waveguides this mirror property remains valid. Let us start from the transversal and longitudinal part of the Maxwell curl equations for the modal field patterns in a general bi-anisotropic waveguide

$$\begin{aligned}\nabla_t\times\mathbf{E}_t&\\ =-j\omega\mu_{zz}H_z\mathbf{u}_z-j\omega\zeta_{zz}E_z\mathbf{u}_z-j\omega\boldsymbol{\mu}_{zt}\cdot\mathbf{H}_t&\\ -j\omega\boldsymbol{\zeta}_{zt}\cdot\mathbf{E}_t,&\end{aligned} \quad (15)$$

$$\begin{aligned}-\gamma\mathbf{u}_z\times\mathbf{E}_t+\nabla_t\times E_z\mathbf{u}_z&\\ =-j\omega\overline{\overline{\mu}}_{tt}\cdot\mathbf{H}_t-j\omega\overline{\overline{\zeta}}_{tt}\cdot\mathbf{E}_t-j\omega\boldsymbol{\mu}_{tz}H_z&\\ -j\omega\boldsymbol{\zeta}_{tz}E_z,&\end{aligned} \quad (16)$$

$$\begin{aligned}\nabla_t\times\mathbf{H}_t&\\ =j\omega\epsilon_{zz}E_z\mathbf{u}_z+j\omega\xi_{zz}H_z\mathbf{u}_z+j\omega\boldsymbol{\epsilon}_{zt}\cdot\mathbf{E}_t&\\ +j\omega\boldsymbol{\xi}_{zt}\cdot\mathbf{H}_t,&\end{aligned} \quad (17)$$

$$\begin{aligned}-\gamma\mathbf{u}_z\times\mathbf{H}_t+\nabla_t\times H_z\mathbf{u}_z&\\ =j\omega\overline{\overline{\epsilon}}_{tt}\cdot\mathbf{E}_t+j\omega\overline{\overline{\xi}}_{tt}\cdot\mathbf{H}_t+j\omega\boldsymbol{\epsilon}_{tz}E_z&\\ +j\omega\boldsymbol{\xi}_{tz}H_z.&\end{aligned} \quad (18)$$

The material dyadics are decomposed as

$$\overline{\overline{\alpha}}=\alpha_{zz}\mathbf{u}_z\mathbf{u}_z+\mathbf{u}_z\boldsymbol{\alpha}_{zt}+\boldsymbol{\alpha}_{tz}\mathbf{u}_z+\overline{\overline{\alpha}}_{tt} \quad (19)$$

with α equal to ϵ, μ, ζ, or ξ. If we insert the mirror transformation formulas in these equations then the same equations are obtained only for materials for which

$$\boldsymbol{\epsilon}_{tz}=\boldsymbol{\epsilon}_{zt}=0 \quad (20)$$

$$\boldsymbol{\mu}_{tz}=\boldsymbol{\mu}_{zt}=0 \quad (21)$$

$$\overline{\overline{\zeta}}_{tt}=\overline{\overline{\xi}}_{tt}=0 \quad (22)$$

and

$$\zeta_{zz}=\xi_{zz}=0. \quad (23)$$

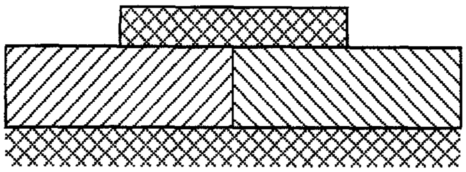

Fig. 1. A microstrip line on an anisotropic substrate with a symmetry line in its cross-section. The hatch-direction indicates the direction of anisotropy. Cross-hatched regions are perfect conductors.

It is clear that these material conditions have nothing whatsoever to do with the reciprocity conditions. In particular this means that certain nonreciprocal waveguides are mirroring and conversely that the modal field patterns of oppositely propagating modes in certain reciprocal waveguides are not just mirror images. An important special case of such a reciprocal waveguide is a waveguide consisting of reciprocal bi-isotropic, i.e., chiral, material. We will call a waveguide which satisfies the conditions (20)–(23) a mirroring waveguide. For each nonmirroring waveguide there exists a waveguide which is a mutually mirror waveguide. The material parameters, indicated by a hat, of this waveguide are given by

$$\begin{aligned}\hat{\overline{\overline{\epsilon}}} &= \hat{\epsilon}_{zz}\mathbf{u}_z\mathbf{u}_z + \mathbf{u}_z\hat{\boldsymbol{\epsilon}}_{zt} + \hat{\boldsymbol{\epsilon}}_{tz}\mathbf{u}_z + \hat{\overline{\overline{\epsilon}}}_{tt}\\ &= \epsilon_{zz}\mathbf{u}_z\mathbf{u}_z - \mathbf{u}_z\boldsymbol{\epsilon}_{zt} - \boldsymbol{\epsilon}_{tz}\mathbf{u}_z + \overline{\overline{\epsilon}}_{tt},\end{aligned} \tag{24}$$

$$\begin{aligned}\hat{\overline{\overline{\mu}}} &= \hat{\mu}_{zz}\mathbf{u}_z\mathbf{u}_z + \mathbf{u}_z\hat{\boldsymbol{\mu}}_{zt} + \hat{\boldsymbol{\mu}}_{tz}\mathbf{u}_z + \hat{\overline{\overline{\mu}}}_{tt}\\ &= \mu_{zz}\mathbf{u}_z\mathbf{u}_z - \mathbf{u}_z\boldsymbol{\mu}_{zt} - \boldsymbol{\mu}_{tz}\mathbf{u}_z + \overline{\overline{\mu}}_{tt},\end{aligned} \tag{25}$$

$$\begin{aligned}\hat{\overline{\overline{\xi}}} &= \hat{\xi}_{zz}\mathbf{u}_z\mathbf{u}_z + \mathbf{u}_z\hat{\boldsymbol{\xi}}_{zt} + \hat{\boldsymbol{\xi}}_{tz}\mathbf{u}_z + \hat{\overline{\overline{\xi}}}_{tt}\\ &= -\xi_{zz}\mathbf{u}_z\mathbf{u}_z + \mathbf{u}_z\boldsymbol{\xi}_{zt} + \boldsymbol{\xi}_{tz}\mathbf{u}_z - \overline{\overline{\xi}}_{tt},\end{aligned} \tag{26}$$

$$\begin{aligned}\hat{\overline{\overline{\zeta}}} &= \hat{\zeta}_{zz}\mathbf{u}_z\mathbf{u}_z + \mathbf{u}_z\hat{\boldsymbol{\zeta}}_{zt} + \hat{\boldsymbol{\zeta}}_{tz}\mathbf{u}_z + \hat{\overline{\overline{\zeta}}}_{tt}\\ &= -\zeta_{zz}\mathbf{u}_z\mathbf{u}_z + \mathbf{u}_z\boldsymbol{\zeta}_{zt} + \boldsymbol{\zeta}_{tz}\mathbf{u}_z - \overline{\overline{\zeta}}_{tt}.\end{aligned} \tag{27}$$

Again, when these material parameters coincide with the material parameters of the adjoint waveguide for a nonreciprocal waveguide then there is a mirror relation between a mode propagating in the original waveguide and the corresponding mode propagating in opposite direction in the adjoint waveguide.

Sometimes there exists a simple relation between modes propagating in opposite directions due to symmetry in the geometry of the waveguide. For example when the geometry contains a symmetry axis in its cross-section [7]. If one just turns around the waveguide by 180° along this axis, chosen in one particular cross-section, the same waveguide is obtained. This means that the oppositely propagating modes are related to each other by the symmetry of the waveguide. Fig. 1 shows and example of such a waveguide consisting of a microstrip line. This means also that nonreciprocal bi-anisotropic waveguides become bi-directional as soon as they have a symmetry axis in their cross-section. Similar conclusions can be drawn when the cross-section of the waveguide contains a symmetry center.

One could ask if there exists a natural relation between the field patterns of modes propagating in opposite direction with the same propagation constant in the adjoint waveguide. The answer is that one can indeed construct such a relation but that it is far from trivial. For a solution $[\gamma_a, \mathbf{E}_{t,a}, \mathbf{H}_{t,a}]$ we can formally write the eigensystem (15)–(18), by eliminating the z-components, as

$$\gamma_a \begin{pmatrix}\mathbf{E}_{t,a}\\ \mathbf{H}_{t,a}\end{pmatrix} = \begin{pmatrix}\overline{\overline{L}}_{11} & \overline{\overline{L}}_{12}\\ \overline{\overline{L}}_{21} & \overline{\overline{L}}_{22}\end{pmatrix}\begin{pmatrix}\mathbf{E}_{t,a}\\ \mathbf{H}_{t,a}\end{pmatrix} \tag{28}$$

where $\overline{\overline{L}}_{ij}$, $(i,\, j = 1,\, 2)$ are complicated two-dimensional (2-D) dyadic vector operators. For explicit expressions for chiral materials we refer to Appendix B. A solution $[\tilde{\gamma}_b,\, \tilde{\mathbf{E}}_{t,b},\, \tilde{\mathbf{H}}_{t,b}]$ of the adjoint waveguide satisfies

$$\tilde{\gamma}_b \begin{pmatrix}\tilde{\mathbf{E}}_{t,b}\\ \tilde{\mathbf{H}}_{t,b}\end{pmatrix} = \begin{pmatrix}\tilde{\overline{\overline{L}}}_{11} & \tilde{\overline{\overline{L}}}_{12}\\ \tilde{\overline{\overline{L}}}_{21} & \tilde{\overline{\overline{L}}}_{22}\end{pmatrix}\begin{pmatrix}\tilde{\mathbf{E}}_{t,b}\\ \tilde{\mathbf{H}}_{t,b}\end{pmatrix} \tag{29}$$

where the operators $\tilde{\overline{\overline{L}}}_{ij}$ are obtained from the operators $\overline{\overline{L}}_{ij}$ by replacing the material parameters with the adjoint material parameters (2)–(5). Now we impose a linear operator relation between each solution $[\gamma_a, \mathbf{E}_{t,a}, \mathbf{H}_{t,a}]$ in the original waveguide and the corresponding solution $[\tilde{\gamma}_b = -\gamma_a,\, \tilde{\mathbf{E}}_{t,b},\, \tilde{\mathbf{H}}_{t,b}]$ in the adjoint waveguide

$$\begin{pmatrix}\tilde{\mathbf{E}}_{t,b}\\ \tilde{\mathbf{H}}_{t,b}\end{pmatrix} = \begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix}\begin{pmatrix}\mathbf{E}_{t,a}\\ \mathbf{H}_{t,a}\end{pmatrix} \tag{30}$$

where $\overline{\overline{A}}$, $\overline{\overline{B}}$, $\overline{\overline{C}}$, and $\overline{\overline{D}}$ are linear dyadic operators. If one inserts this in both sides of the adjoint eigensystem (29) and demands that the result reduces to the original eigensystem then one finds

$$\begin{aligned}&\begin{pmatrix}\tilde{\overline{\overline{L}}}_{11} & \tilde{\overline{\overline{L}}}_{12}\\ \tilde{\overline{\overline{L}}}_{21} & \tilde{\overline{\overline{L}}}_{22}\end{pmatrix}\begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix}\begin{pmatrix}\mathbf{E}_{t,a}\\ \mathbf{H}_{t,a}\end{pmatrix} =\\ &-\gamma_a\begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix}\begin{pmatrix}\mathbf{E}_{t,a}\\ \mathbf{H}_{t,a}\end{pmatrix}.\end{aligned} \tag{31}$$

If this has to be valid for all eigenmodes in the original waveguide then this means that we can write the following operator equation

$$\begin{aligned}&\begin{pmatrix}\tilde{\overline{\overline{L}}}_{11} & \tilde{\overline{\overline{L}}}_{12}\\ \tilde{\overline{\overline{L}}}_{21} & \tilde{\overline{\overline{L}}}_{22}\end{pmatrix}\begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix} =\\ &-\begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix}\begin{pmatrix}\overline{\overline{L}}_{11} & \overline{\overline{L}}_{12}\\ \overline{\overline{L}}_{21} & \overline{\overline{L}}_{22}\end{pmatrix}.\end{aligned} \tag{32}$$

We also demand that the linear transformation (30) is an involution

$$\begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix}\begin{pmatrix}\overline{\overline{A}} & \overline{\overline{B}}\\ \overline{\overline{C}} & \overline{\overline{D}}\end{pmatrix} = \begin{pmatrix}\overline{\overline{U}} & \overline{\overline{0}}\\ 0 & \overline{\overline{U}}\end{pmatrix} \tag{33}$$

where the dyadic operators $\overline{\overline{U}}$ are unit operators. The solution of this set of operator equations (32) and (33) is difficult. In Appendix B the way to construct a solution is indicated for the special case of a chiral waveguide.

IV. Generalized Transmission Lines

A coupled set of N generalized transmission lines is described by the generalized telegrapher's equations

$$\frac{d\mathbf{v}(z)}{dz} = -\overline{\overline{P}}\mathbf{v}(z) - \overline{\overline{Z}}\mathbf{i}(z), \tag{34}$$

$$\frac{d\mathbf{i}(z)}{dz} = -\overline{\overline{Y}}\mathbf{v}(z) - \overline{\overline{Q}}\mathbf{i}(z) \tag{35}$$

where $\mathbf{v}(z)$ and $\mathbf{i}(z)$ are column matrices with the N voltages and currents of the different lines. The $N \times N$ matrices $\overline{\overline{Z}}$, $\overline{\overline{Y}}$, $\overline{\overline{P}}$, and $\overline{\overline{Q}}$ are the transmission line parameters. The properties of losslessness and reciprocity are discussed in [8]. In particular it was shown that these transmission lines are reciprocal when $\overline{\overline{Z}} = \overline{\overline{Z}}^T$, $\overline{\overline{Y}} = \overline{\overline{Y}}^T$, and $\overline{\overline{P}} = -\overline{\overline{Q}}^T$.

Suppose that $[\mathbf{v}_a(z), \mathbf{i}_a(z)]$ is a solution of the transmission line (34) and (35) and suppose that $[\tilde{\mathbf{v}}_b(z), \tilde{\mathbf{i}}_b(z)]$ is an independent solution of the adjoint transmission line. The adjoint transmission line has parameters given by

$$\tilde{\overline{\overline{Z}}} = \overline{\overline{Z}}^T \tag{36}$$

$$\tilde{\overline{\overline{Y}}} = \overline{\overline{Y}}^T \tag{37}$$

$$\tilde{\overline{\overline{P}}} = -\overline{\overline{Q}}^T \tag{38}$$

and

$$\tilde{\overline{\overline{Q}}} = -\overline{\overline{P}}^T. \tag{39}$$

It is now easy to verify that the following relation holds between the a and b solution in each source free region of the original and adjoint transmission line

$$\frac{d}{dz}\left[\mathbf{i}_a{}^T(z)\tilde{\mathbf{v}}_b(z) - \tilde{\mathbf{i}}_b{}^T(z)\mathbf{v}_a(z)\right] = 0. \tag{40}$$

This is the equivalent, although in differential form, for transmission lines of the Lorentz reciprocity relation (1) for waveguides.

Assume that the a solution is a mode with propagation constant γ_a in the set of transmission lines described by

$$\mathbf{v}_a(z) = \mathbf{V}_a \exp(-\gamma_a z), \tag{41}$$

$$\mathbf{i}_a(z) = \mathbf{I}_a \exp(-\gamma_a z) \tag{42}$$

where $\mathbf{V}_a$ and $\mathbf{I}_a$ are the modal voltage and current patterns. Similarly let the b solution be a mode propagating in the adjoint set of transmission lines with propagation constant $\tilde{\gamma}_b$. Expanding (40) for these modal solutions and integrating from $z = 0$ to $z = L$ along the transmission lines results in

$$\{1 - \exp[-(\gamma_a + \tilde{\gamma}_b)L]\}(\mathbf{I}_a^T\tilde{\mathbf{V}}_b - \tilde{\mathbf{I}}_b^T\mathbf{V}_a) = 0. \tag{43}$$

This relation is the bi-orthogonality relation for modes in generalized transmission lines. If $\gamma_a \neq -\tilde{\gamma}_b$ then the second factor in (43) has to be zero.

If the modal representations (41) and (42) are inserted in (34) and (35) then the following linear eigenvalue system is obtained for the modes

$$-\gamma_a \begin{pmatrix} \mathbf{V}_a \\ \mathbf{I}_a \end{pmatrix} = -\begin{pmatrix} \overline{\overline{P}} & \overline{\overline{Z}} \\ \overline{\overline{Y}} & \overline{\overline{Q}} \end{pmatrix} \begin{pmatrix} \mathbf{V}_a \\ \mathbf{I}_a \end{pmatrix}. \tag{44}$$

The corresponding eigenvalue equation is given by

$$\det \begin{pmatrix} \overline{\overline{P}} - \gamma_a\overline{\overline{U}} & \overline{\overline{Z}} \\ \overline{\overline{Y}} & \overline{\overline{Q}} - \gamma_a\overline{\overline{U}} \end{pmatrix} = 0 \tag{45}$$

where $\overline{\overline{U}}$ is the $N \times N$ unit matrix. Now by some matrix manipulations it is easy to show that (45) also implies that

$$\det \begin{pmatrix} -\overline{\overline{Q}}^T + \gamma_a\overline{\overline{U}} & \overline{\overline{Z}}^T \\ \overline{\overline{Y}}^T & -\overline{\overline{P}}^T + \gamma_a\overline{\overline{U}} \end{pmatrix} = 0. \tag{46}$$

This proves that the original and adjoint set of transmission lines are mutual bi-directional and in particular, corresponding to the result in [8], that reciprocal generalized transmission lines are bi-directional.

In general it can be checked, as for the waveguides, that the modal voltages and currents of a mode and the voltages and currents of the corresponding mode in the adjoint waveguide with the same propagation constant but propagating in the opposite direction are not just mirror images. They are mirror images only under the conditions that

$$\overline{\overline{Z}} = \overline{\overline{Z}}^T, \tag{47}$$

$$\overline{\overline{Y}} = \overline{\overline{Y}}^T \tag{48}$$

and

$$\overline{\overline{Q}} = \overline{\overline{P}}^T. \tag{49}$$

For reciprocal transmission lines this means that $\overline{\overline{P}}$ and $\overline{\overline{Q}}$ vanish or that the transmission lines reduce to ordinary transmission lines.

Again, the question arises if one can construct a natural linear relation between the modal voltages and currents of these oppositely propagating modes when the relations (47)–(49) are not satisfied. This relation can be written as

$$\begin{pmatrix} \tilde{\mathbf{V}}_b \\ \tilde{\mathbf{I}}_b \end{pmatrix} = \begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} \begin{pmatrix} \mathbf{V}_a \\ \mathbf{I}_a \end{pmatrix} \tag{50}$$

for all modes a and corresponding modes b with $\tilde{\gamma}_b = -\gamma_a$. If one inserts this in both sides of the adjoint eigenvalue system of (44) and demand that this reduces to the original eigenvalue system (44) then one finds

$$\begin{pmatrix} -\overline{\overline{Q}}^T & \overline{\overline{Z}}^T \\ \overline{\overline{Y}}^T & -\overline{\overline{P}}^T \end{pmatrix} \begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} \begin{pmatrix} \mathbf{V}_a \\ \mathbf{I}_a \end{pmatrix} = -\gamma_a \begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} \begin{pmatrix} \mathbf{V}_a \\ \mathbf{I}_a \end{pmatrix} \tag{51}$$

for all modes a. Using the diagonalization of the matrix

$$\begin{pmatrix} \overline{\overline{P}} & \overline{\overline{Z}} \\ \overline{\overline{Y}} & \overline{\overline{Q}} \end{pmatrix} \tag{52}$$

it is easy to show that (51) can be written compactly as

$$\begin{pmatrix} -\overline{\overline{Q}}^T & \overline{\overline{Z}}^T \\ \overline{\overline{Y}}^T & -\overline{\overline{P}}^T \end{pmatrix} \begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} = -\begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} \begin{pmatrix} \overline{\overline{P}} & \overline{\overline{Z}} \\ \overline{\overline{Y}} & \overline{\overline{Q}} \end{pmatrix}. \tag{53}$$

Further we demand that the transformation (50) is an involution, i.e.,

$$\begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} \begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} = \begin{pmatrix} \overline{\overline{U}} & 0 \\ 0 & \overline{\overline{U}} \end{pmatrix}. \tag{54}$$

The analytical solution of the system of (53) and (54) is rather cumbersome and is discussed in Appendix C. Let us look here at two special cases.

The solution for a single generalized transmission line is easily found

$$A = -D = \frac{Y+Z}{\sqrt{(Y+Z)^2 + (P-Q)^2}} \quad (55)$$

and

$$B = C = -\frac{P-Q}{\sqrt{(Y+Z)^2 + (P-Q)^2}} \quad (56)$$

where we had the freedom to choose $B = C$. Remark that when $P = Q = 0$ the mirror transformation for the modal currents and voltages is recovered.

As second special case we consider the perturbation solution for a generalized transmission line with small parameters $\overline{\overline{P}}$ and $\overline{\overline{Q}}$, i.e., with $\overline{\overline{P}} = \delta\overline{\overline{P}}$ and $\overline{\overline{Q}} = \delta\overline{\overline{Q}}$ and $\overline{\overline{y}}$ and $\overline{\overline{z}}$ symmetries. Since the solution for $\overline{\overline{P}} = \overline{\overline{Q}} = 0$ is given by $\overline{\overline{A}} = -\overline{\overline{D}} = \overline{\overline{U}}$ and $\overline{\overline{B}} = \overline{\overline{C}} = 0$ the solution for small $\delta\overline{\overline{P}}$ and $\delta\overline{\overline{Q}}$ will be of the form $\overline{\overline{A}} = \overline{\overline{U}} + \delta\overline{\overline{A}}$, $\overline{\overline{D}} = -\overline{\overline{U}} + \delta\overline{\overline{D}}$, $\overline{\overline{B}} = \delta\overline{\overline{B}}$ and $\overline{\overline{C}} = \delta\overline{\overline{C}}$. If we insert these forms in (54) and keep only first order terms one finds already that $\delta\overline{\overline{A}} = \delta\overline{\overline{D}} = 0$. Equation (53) yields

$$\delta\overline{\overline{Q}} - \delta\overline{\overline{P}} = \overline{\overline{Y}}\,\delta\overline{\overline{B}} + \delta\overline{\overline{C}}\,\overline{\overline{Z}} \quad (57)$$

and

$$\delta\overline{\overline{Q}} - \delta\overline{\overline{P}}^T = \overline{\overline{Y}}\,\delta\overline{\overline{B}}^T + \delta\overline{\overline{C}}^T\overline{\overline{Z}}. \quad (58)$$

By adding and subtracting these equations one concludes that the asymmetric parts of $\delta\overline{\overline{B}}$ and $\delta\overline{\overline{C}}$ vanish and that the symmetric parts, indicated by a subscript "S," satisfy

$$\delta\overline{\overline{Q}} - \delta\overline{\overline{P}}^T = \delta\overline{\overline{C}}_S\overline{\overline{Z}} + \overline{\overline{Y}}\,\delta\overline{\overline{B}}_S. \quad (59)$$

This is a linear set of N^2 equations with $N^2 + N$ unknowns. The N degrees of freedom left can be chosen at will, for example one could choose the same eigenvalues for $\delta\overline{\overline{B}}$ and $\delta\overline{\overline{C}}$.

From the general case discussed in Appendix C we can draw the following conclusions. The system (53) and (54) has still N degrees of freedom left, the matrices $\overline{\overline{B}}$ and $\overline{\overline{C}}$ are symmetric matrices and $\overline{\overline{D}} = -\overline{\overline{A}}^T$.

V. Transmission Line Models

By comparing previous sections a large analogy between the modes in a waveguide and the modes in a coupled set of transmission lines becomes apparent. Our aim is to construct a transmission line model which as good as possible represents the propagation of a certain number of modes in a waveguide. In [3], transmission line models were constructed for modes in reciprocal mirroring anisotropic waveguides. Now we want to extend these models to general not necessary reciprocal bi-anisotropic waveguides. To understand the full background of these transmission line models the reader is referred to [3]. The main point in [3] was that transmission line models should be based on reciprocity rather than on power assumptions. Indeed it was shown in [3], that power based transmission line models for lossy reciprocal waveguides are nonreciprocal which is intolerable.

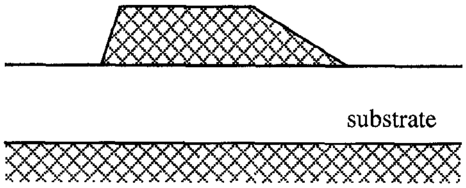

Fig. 2. A microstrip line with asymmetric trapezoidal cross-section.

In [3], different possible transmission line models were constructed such as RI-models, RV-models and more fundamental models taking into account the true interaction with the generator and receiver. Here we will concentrate ourselves on RI-models, i.e., transmission line models where the currents in the transmission lines correspond to currents in the waveguide. The other models are easily deduced from the RI-model as in [3]. Although we will concentrate on the RI-model for multiconductor waveguides, such as the microstrip line, the results can also be used for optical and dielectric waveguides as discussed in [3].

There are a number of criteria which have to be satisfied for the transmission line models which we will introduce here. First, when the waveguide becomes reciprocal, mirroring and anisotropic the model has to reduce to the classical transmission line model of [3]. Second, a transmission line model for a reciprocal waveguide should be reciprocal. Third, at low frequencies, i.e., in the so-called quasi-TEM limit, the generalized transmission line model should reduce to the generalized transmission lines which follow then immediately from the Maxwell equations as shown in [8].

First, we will construct a single generalized transmission line model for two oppositely propagating modes in a bi-directional waveguide. Next, this will be generalized to two modes in a general nonreciprocal waveguide. Finally, coupled generalized transmission line models for a number of modes in general nonreciprocal waveguides are discussed.

A. Single Transmission Line Model for a Bi-Directional Waveguide

Consider two modes with opposite propagation constants in a waveguide. In particular this could be two modes in a reciprocal waveguide such as an asymmetric microstrip line (Fig. 2) placed on a reciprocal bi-anisotropic substrate. The fields of both modes are not just mirror images. This means for example that the longitudinal currents on the microstrip line for both modes are not just opposite.

We represent the propagation of these modes by a single reciprocal generalized transmission line

$$\frac{dv(z)}{dz} = -Pv(z) - Zi(z), \quad (60)$$

$$\frac{di(z)}{dz} = -Yv(z) + Pi(z). \quad (61)$$

The quantities P, Y, and Z are the unknowns in this problem. The modal solutions of this coupled set of equations are found from the eigenvalue problem

$$-\gamma V = -PV - ZI \quad (62)$$

$$-\gamma I = -YV + PI. \quad (63)$$

The solutions of this eigenvalue problem are given by

$$\gamma_\pm = \pm\sqrt{ZY + P^2} \tag{64}$$

and

$$V_\pm = \frac{Z}{\gamma_\pm - P} I_\pm. \tag{65}$$

From (43) it follows that $I_+V_- - I_-V_+$ does not vanish and can be used to normalize the eigenmodes

$$\tfrac{1}{4}(I_+V_- - I_-V_+) = 1. \tag{66}$$

The factor 1/4 in front of this expression is chosen to be in accordance with [3]. If one eliminates $V_\pm$ from this equation using (65) and one solves for Y one finds

$$Y = -\frac{\gamma_+ I_- I_+}{2}. \tag{67}$$

Similarly if one eliminates $I_\pm$ and solves for Z one finds

$$Z = \frac{\gamma_+ V_- V_+}{2}. \tag{68}$$

Now we have to make an identification between the modes in the waveguide and the modes in the circuit model. First, we demand that the propagation constants $\pm\gamma$ in the transmission line model are equal to those of the modes in the waveguide. Second, we demand that the currents $I_\pm$ are equal to the currents on some conductors, for example the longitudinal currents on the microstrip, of the waveguide. In order to make this identification the modes in the waveguide and the transmission line should be normalized in the same way. Since we already used (66) as a normalization we have to impose the equivalent normalization for the modes in the waveguide

$$\tfrac{1}{4}\int_S [\mathbf{H}_{t,+}(x,y) \times \mathbf{E}_{t,-}(x,y) - \mathbf{H}_{t,-}(x,y) \times \mathbf{E}_{t,+}(x,y)].\mathbf{u}_z\, dS = 1. \tag{69}$$

Equation (66) (or equivalently (69) in the waveguide) does not fully determine the amplitudes of the eigenmodes. Indeed there is only one equation specifying the amplitudes of two modes. In [3], the extra needed equation arose automatically from the fact that both modes were mirror images. In this case, we could use the relations derived in Section III to link the modes but let us first look at a few more easy and practical techniques to specify the amplitudes. So in essence we need one extra equation besides (66) to normalize the modes.

A first possibility would be to impose that

$$\tfrac{1}{2}(I_\pm V_\mp) = \pm 1. \tag{70}$$

Note that this includes (66) and that this is compatible with the normalization in [3]. The corresponding normalization in the waveguide would then be

$$\tfrac{1}{2}\int_S [\mathbf{H}_{t,\pm}(x,y) \times \mathbf{E}_{t,\mp}(x,y)].\mathbf{u}_z\, dS = \pm 1. \tag{71}$$

This fixes the amplitudes of both modes in the waveguide and in the transmission line. In the RI-model we can now demand that the currents in the transmission line are the same as for example on the microstrip line. Let us express the quantities Y, Z, and P as a function of the known quantities γ_+, I_+, and I_-. Equation (67) immediately yields Y as a function of these known quantities. The impedance Z is obtained from (68) by elimination of the voltages using (70)

$$Z = -\frac{2\gamma_+}{I_+ I_-}. \tag{72}$$

Finally the quantity P follows from (65), using (70)

$$P = \gamma_+ + \frac{Z I_+ I_-}{2} = 0. \tag{73}$$

The remarkable result is obtained that $P = 0$ under this normalization.

Let us look at a second possibility by taking as second normalization the quantity

$$I_+V_+ + I_-V_- = 0. \tag{74}$$

This normalization is again compatible with [3], i.e., for mirroring modes this relation is automatically satisfied. The corresponding normalization for waveguides is

$$\int_S [\mathbf{H}_{t,+}(x,y) \times \mathbf{E}_{t,+}(x,y) + \mathbf{H}_{t,-}(x,y) \times \mathbf{E}_{t,-}(x,y)].\mathbf{u}_z\, dS = 0. \tag{75}$$

The impedance Z and the quantity P are now given by

$$Z = -\frac{8\gamma_+ I_- I_+}{(I_+^2 + I_-^2)^2} \tag{76}$$

and

$$P = \frac{I_-^2 - I_+^2}{I_+^2 + I_-^2}\gamma_+. \tag{77}$$

Both (70) and (74) do not automatically lead to the generalized transmission line model of [8] for low frequencies in the quasi-TEM limit. A third, and most simple, extra normalization which will yield a transmission line model compatible with [8] is

$$V_+ = V_-. \tag{78}$$

In the waveguide this means that we impose that

$$\int_l \mathbf{E}_{t,+}(x,y).d\mathbf{l} = \int_l \mathbf{E}_{t,-}(x,y).d\mathbf{l} \tag{79}$$

where l is a chosen path in the cross-section, typically connecting the microstrip with the substrate. Each path will yield a different circuit model but in the quasi-TEM limit the integral becomes path-independent and will reduce to the unique quasi-TEM model in [8]. The transmission line parameters are now given by

$$Z = \frac{8\gamma_+}{(I_+ - I_-)^2} \tag{80}$$

and

$$P = \frac{I_+ + I_-}{I_- - I_+}\gamma_+. \tag{81}$$

As fourth possibility we can use the link between the modes in the transmission line derived in Section IV. This means that $[V_+, I_+]$ and $[V_-, I_-]$ are related by

$$\begin{pmatrix} V_- \\ I_- \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_+ \\ I_+ \end{pmatrix} \tag{82}$$

with A, B, C, and D given by (55) and (56). If we solve this set of equations for V_+ and V_- one finds

$$V_\pm = -\frac{\sqrt{(Y+Z)^2+4P^2}}{2P} I_\mp - \frac{Y+Z}{2P} I_\pm. \tag{83}$$

If these expressions are inserted in (66) one finds that

$$P = \frac{I_-^2 - I_+^2}{2\sqrt{16-(I_-^2-I_+^2)^2}} (Y+Z). \tag{84}$$

If on the other hand the expressions (83) are inserted in the impedance (68) using (84) then the final expression for Z is obtained as shown in (85) at the bottom of the page, P follows then from (84). The major advantage of the transmission line model derived in this way is that it is the natural extension of the transmission line model derived in [3] to nonmirroring modes. The drawback is that the modes in the waveguide have to be normalized in the same way as in the transmission line or that these modes have to be linked as described in Section III where it was shown that this is far from trivial.

B. Single Transmission Line Model for a Nonbidirectional Waveguide

In this section, we drop the assumption that the two modes have opposite propagation constants. The modes are modes in a nonreciprocal waveguide, for example two modes in an asymmetric microstrip line (Fig. 2) placed on a nonreciprocal bi-anisotropic substrate. To represent these modes we need a nonreciprocal generalized transmission line

$$\frac{dv(z)}{dz} = -Pv(z) - Zi(z) \tag{86}$$

$$\frac{di(z)}{dz} = -Yv(z) - Qi(z). \tag{87}$$

Now there are four unknowns P, Z, Y, and Q. If the modal representations for the voltage and current are inserted in these equations the following expression for the propagation constants is obtained

$$\gamma_\pm = \frac{P+Q \pm \sqrt{(P+Q)^2 - 4(PQ-YZ)}}{2} \tag{88}$$

and the relation between the modal voltage and current is given by

$$V_\pm = \frac{Z}{\gamma_\pm - P} I_\pm. \tag{89}$$

To apply the reciprocity theorem (43) we also need the adjoint transmission line described by

$$\frac{d\tilde{v}(z)}{dz} = Q\tilde{v}(z) - Z\tilde{i}(z), \tag{90}$$

$$\frac{d\tilde{i}(z)}{dz} = -Y\tilde{v}(z) + P\tilde{i}(z). \tag{91}$$

The propagation constants for the adjoint system satisfy

$$\tilde{\gamma}_\pm = \frac{-(P+Q) \pm \sqrt{(P+Q)^2 - 4(PQ-YZ)}}{2} \tag{92}$$

and the relation between modal voltage and current now becomes

$$\tilde{V}_\pm = \frac{Z}{\tilde{\gamma}_\pm + Q} \tilde{I}_\pm. \tag{93}$$

From the reciprocity theorem (43) we now obtain four relations between the modal currents and voltages for the original and adjoint transmission line

$$\tfrac{1}{4}(I_+\tilde{V}_- - \tilde{I}_-V_+) = 1, \tag{94}$$

$$\tfrac{1}{4}(\tilde{I}_+V_- - I_-\tilde{V}_+) = 1, \tag{95}$$

$$I_+\tilde{V}_+ - \tilde{I}_+V_+ = 0, \tag{96}$$

$$I_-\tilde{V}_- - \tilde{I}_-V_- = 0 \tag{97}$$

where the two first equations already are used to normalize the modes. Here and in future normalization equations we will not explicitly give the corresponding expressions for the normalizations in the waveguide, these are easily deduced by the reader. If one eliminates V_+ and $\tilde{V}_-$ from (94) using (89) and (93) and taking into account that $\tilde{\gamma}_- = -\gamma_+$, that $P+Q = \gamma_+ + \gamma_-$ and that $(\gamma_+ - P)(\gamma_+ - Q) = YZ$, one can show that

$$Y = -\frac{(\gamma_+ - \gamma_-)\tilde{I}_-I_+}{4}. \tag{98}$$

If conversely $\tilde{I}_-$ and I_+ are eliminated from (94) the following expression for the impedance Z is obtained

$$Z = \frac{(\gamma_+ - \gamma_-)\tilde{V}_-V_+}{4}. \tag{99}$$

Since there are four modes into play now, we need four normalization equations to fix the amplitudes of the modes, this means two extra in addition to (94) and (95). These normalization expressions will allow us to express Y, Z, P, and Q as a function of the known quantities $I_\pm$ and $\gamma_\pm$.

First, we consider the generalization of the normalization condition (78) to the nonreciprocal case. This gives the following two extra normalization conditions

$$V_\pm = \tilde{V}_\mp. \tag{100}$$

Using these in combination with (94) and (95) results after some steps in

$$V_+ = V_- = \tilde{V}_+ = \tilde{V}_- = \frac{4}{I_+ - I_-} \tag{101}$$

and

$$\tilde{I}_\pm = I_\pm. \tag{102}$$

$$Z = \frac{\gamma_+}{2} \frac{4\sqrt{16-(I_-^2-I_+^2)^2}(I_-^2+I_+^2) + [32-(I_-^2-I_+^2)^2]I_-I_+}{(I_-^2-I_+^2)^2} \tag{85}$$

These results immediately give us the final expressions for the admittance Y (98) and impedance Z (99)

$$Y = -\frac{(\gamma_+ - \gamma_-)I_- I_+}{4} \tag{103}$$

$$Z = \frac{4(\gamma_+ - \gamma_-)}{(I_+ - I_-)^2} \tag{104}$$

and from (89) and (93) then follows

$$P = \frac{\gamma_- I_+ - \gamma_+ I_-}{I_+ - I_-} \tag{105}$$

and

$$Q = \frac{\gamma_+ I_+ - \gamma_- I_-}{I_+ - I_-} \tag{106}$$

respectively.

As second more natural, but less practical, extra set of normalization conditions we use the natural relations between the modes in the original and adjoint transmission line, discussed in Section IV. We will restrict ourselves to the results. First of all it follows that

$$\tilde{I}_\pm = I_\pm \tag{107}$$

and

$$\tilde{V}_\pm = V_\pm. \tag{108}$$

This means that Y follows from (98) by removing the tilde. The impedance is given by (109) shown at the bottom of the page, and P and Q follow from the observation that $P + Q = \gamma_+ + \gamma_-$ and that

$$P - Q = \frac{I_-^2 - I_+^2}{\sqrt{16 - (I_-^2 - I_+^2)^2}}(Y + Z). \tag{110}$$

C. Coupled Generalized Transmission Line Models

In [3], it was shown that it is relatively easy to generalize the ordinary single transmission line models for modes in mirroring waveguides to coupled transmission lines. In the case of generalized transmission lines this is no longer trivial as can be guessed from the complications already encountered in the single generalized transmission lines. These complications will not allow us to present closed form expressions for $\overline{\overline{Y}}$, $\overline{\overline{Z}}$, $\overline{\overline{P}}$, and $\overline{\overline{Q}}$ as was possible in [3] for $\overline{\overline{Y}}$ and $\overline{\overline{Z}}$.

Assume N modes propagating in each direction in a reciprocal waveguide. We represent these modes by the coupled set of N generalized reciprocal transmission lines (34) and (35). Let $\overline{\overline{V}}_\pm$ ($\overline{\overline{I}}_\pm$) represent an $N \times N$ matrix with on its columns the N voltages (currents) $\mathbf{V}_{\pm,i}(\mathbf{I}_{\pm,i})$ with $i = 1, \cdots, N$. The propagation constants $\gamma_+ = -\gamma_-$ are grouped in a $N \times N$ diagonal matrix $\overline{\overline{\gamma}}_+ = -\overline{\overline{\gamma}}_-$.

In the RI-model we want to determine the parameter matrices $\overline{\overline{Y}}$, $\overline{\overline{Z}}$, and $\overline{\overline{P}}$ from the knowledge of the eigenvalues $\overline{\overline{\gamma}}_+$ and the currents $\overline{\overline{I}}_\pm$. The $2N^2$ currents in $\overline{\overline{I}}_\pm$ are typically taken equal to the currents on N conductors of N modes propagating in the positive direction and N modes propagating in the negative direction. This requires the normalization of these modes in the waveguide as well as the set of transmission lines. From all this we conclude that there are $2N^2 + N$ unknown quantities to be determined from $2N^2 + N$ given quantities and that we need $2N$ normalization equations for the modes in the transmission lines and the waveguide.

The relation (44) between the modal voltages and currents can then be written as

$$\begin{pmatrix} \overline{\overline{V}}_+ & \overline{\overline{V}}_- \\ \overline{\overline{I}}_+ & \overline{\overline{I}}_- \end{pmatrix} \begin{pmatrix} \overline{\overline{\gamma}}_+ & 0 \\ 0 & \overline{\overline{\gamma}}_- \end{pmatrix} = \begin{pmatrix} \overline{\overline{P}} & \overline{\overline{Z}} \\ \overline{\overline{Y}} & -\overline{\overline{P}}^T \end{pmatrix} \begin{pmatrix} \overline{\overline{V}}_+ & \overline{\overline{V}}_- \\ \overline{\overline{I}}_+ & \overline{\overline{I}}_- \end{pmatrix}. \tag{111}$$

Elimination of the voltages $\overline{\overline{V}}_\pm$ from these equations yields

$$\overline{\overline{Y}}^{-1}(\overline{\overline{I}}_+\overline{\overline{\gamma}}_+ + \overline{\overline{P}}^T\overline{\overline{I}}_+)\overline{\overline{\gamma}}_+ = \overline{\overline{PY}}^{-1}(\overline{\overline{I}}_+\overline{\overline{\gamma}}_+ + \overline{\overline{P}}^T\overline{\overline{I}}_+) + \overline{\overline{ZI}}_+, \tag{112}$$

$$-\overline{\overline{Y}}^{-1}(-\overline{\overline{I}}_-\overline{\overline{\gamma}}_+ + \overline{\overline{P}}^T\overline{\overline{I}}_-)\overline{\overline{\gamma}}_+ = \overline{\overline{PY}}^{-1}(-\overline{\overline{I}}_-\overline{\overline{\gamma}}_+ + \overline{\overline{P}}^T\overline{\overline{I}}_-) + \overline{\overline{ZI}}_-. \tag{113}$$

These equations are not sufficient to determine the parameters $\overline{\overline{P}}$, $\overline{\overline{Y}}$, and $\overline{\overline{Z}}$. We need to add $2N$ normalization equations. The first N normalization equations follow from the reciprocity relation (43)

$$\tfrac{1}{4}(\overline{\overline{I}}_+{}^T\overline{\overline{V}}_- - \overline{\overline{I}}_-{}^T\overline{\overline{V}}_+) = \overline{\overline{U}} \tag{114}$$

where (111) allows elimination of the voltages in this expression. For the other N normalization equations there are again different possibilities. For example one could ask that one voltage of each mode propagating in the positive direction is equal to a voltage of a mode propagating in the negative direction or one could use the natural relation between the modes obtained in Section IV.

Whatever the chosen normalization be, it is not possible to give a general closed form expression for the parameters $\overline{\overline{P}}$, $\overline{\overline{Y}}$, and $\overline{\overline{Z}}$ as a function of $\overline{\overline{\gamma}}_+$ and $\overline{\overline{I}}_\pm$. However, in practice the set (112) and (113) together with the normalization equations can be solved numerically.

Now consider N modes in a nonreciprocal waveguide. These are represented by a coupled set of nonreciprocal transmission lines (34) and (35). All the elements of the $\overline{\overline{Y}}$, $\overline{\overline{Z}}$, $\overline{\overline{P}}$, and $\overline{\overline{Q}}$ matrices are independent and hence there are $4N^2$ unknowns in this problem. In the RI-model the $2N^2$ currents

$$Z = \frac{\gamma_+ - \gamma_-}{4}\,\frac{4\sqrt{16 - (I_-^2 + I_+^2)^2}(I_-^2 + I_+^2) + [32 - (I_-^2 - I_+^2)^2]I_- I_+}{(I_-^2 - I_+^2)^2} \tag{109}$$

$\overline{\overline{I}}_{\pm}$ *and the* $2N$ propagation constants are given quantities following from the waveguide. This means that there are $2N^2 - 2N$ degrees of freedom left. These can be used to impose other equivalences between the waveguide and the transmission lines. For example one could take $2N^2 - 2N$ currents of the adjoint waveguide equal to currents in the adjoint transmission line, these could be all the nondiagonal elements of the $\overline{\overline{I}}_{\pm}$ matrices. Apart from these there are also $4N$ normalization relations needed to fix the amplitudes of the modes in the original and adjoint waveguide and transmission line. $2N$ normalization relations follow from the normalization of the Lorentz reciprocity expressions and $2N$ more follow from additional relations between the eigenmodes such as the natural relations discussed in Section IV.

VI. Conclusion

A detailed investigation was made of the modal propagation in a bi-anisotropic waveguide and a set of coupled generalized transmission lines. In particular, the relation between corresponding modes propagating in opposite directions in the original and adjoint waveguide and transmission line was studied. In the second part a relation between some modes in a waveguide and a set of coupled generalized transmission lines was constructed. In other words a transmission line model, based on reciprocity considerations, was constructed for the propagation in bi-anisotropic waveguides. In particular, explicit expressions were given for the transmission line parameters, in single generalized reciprocal and nonreciprocal transmission lines, as a function of modal quantities in the waveguide.

Appendix A

This appendix contains a glossary of some terms.

Hybrid Waveguide: Waveguide with inhomogeneous cross-section such as an optical fiber or microstrip line. A coaxial cable is not hybrid.

Bidirectional Waveguide (Transmission Line): Waveguide (transmission line) where for each mode propagating in one direction there is another mode propagating in the other direction with the same propagation constant.

Mirroring Waveguide (Transmission Line): Bi-directional waveguide (transmission line) where the modal field patterns (modal currents and voltages) of the two oppositely propagating modes are equal.

Reciprocal Waveguide (Transmission Line): Waveguide (transmission line) with reciprocal material (transmission line) parameters.

Adjoint Waveguide (Transmission Line): Waveguide (transmission line) corresponding to a nonreciprocal waveguide (transmission line) with reciprocal material (transmission line) parameters (2)–(5) and (36)–(39). The original and adjoint waveguide (transmission line) are mutually bi-directional.

Mutually Mirroring Waveguide: Waveguide corresponding to a nonmirroring waveguide with material parameters (24)–(27). For each mode in the original waveguide there is another mode in the mutually mirroring waveguide with mirrored field patterns.

Appendix B

In this appendix more aspects regarding the determination of the dyadic operators relating modes propagating in opposite directions are illustrated for a chiral waveguide. In a chiral waveguide the modal field patterns satisfy the curl equations

$$\nabla_t \times \mathbf{E}_t = -j\omega\mu H_z \mathbf{u}_z + \omega\kappa\sqrt{\epsilon_0\mu_0} E_z \mathbf{u}_z, \quad (115)$$

$$-\gamma \mathbf{u}_z \times \mathbf{E}_t + \nabla_t \times E_z \mathbf{u}_z = -j\omega\mu \mathbf{H}_t + \omega\kappa\sqrt{\epsilon_0\mu_0}\mathbf{E}_t, \quad (116)$$

$$\nabla_t \times \mathbf{H}_t = j\omega\epsilon E_z \mathbf{u}_z + \omega\kappa\sqrt{\epsilon_0\mu_0} H_z \mathbf{u}_z, \quad (117)$$

$$-\gamma \mathbf{u}_z \times \mathbf{H}_t + \nabla_t \times H_z \mathbf{u}_z = j\omega\epsilon \mathbf{E}_t + \omega\kappa\sqrt{\epsilon_0\mu_0}\mathbf{H}_t. \quad (118)$$

If the z-components are eliminated the following expressions for the operators $\overline{\overline{L}}_{ij}$, $(i, j = 1, 2)$ appearing in (28) can be verified

$$\begin{aligned}\overline{\overline{L}}_{11} &= \overline{\overline{L}}_{22} \\ &= \frac{\partial}{\partial x}\frac{\omega\kappa\sqrt{\epsilon_0\mu_0}}{n^2}\frac{\partial}{\partial y}\mathbf{u}_x\mathbf{u}_x \\ &\quad - \left(\frac{\partial}{\partial x}\frac{\omega\kappa\sqrt{\epsilon_0\mu_0}}{n^2}\frac{\partial}{\partial x} - \omega\kappa\sqrt{\epsilon_0\mu_0}\right)\mathbf{u}_x\mathbf{u}_y \\ &\quad + \left(\frac{\partial}{\partial y}\frac{\omega\kappa\sqrt{\epsilon_0\mu_0}}{n^2}\frac{\partial}{\partial y} - \omega\kappa\sqrt{\epsilon_0\mu_0}\right)\mathbf{u}_y\mathbf{u}_x \\ &\quad - \frac{\partial}{\partial y}\frac{\omega\kappa\sqrt{\epsilon_0\mu_0}}{n^2}\frac{\partial}{\partial x}\mathbf{u}_y\mathbf{u}_y,\end{aligned} \quad (119)$$

$$\begin{aligned}\overline{\overline{L}}_{12} &= \frac{\partial}{\partial x}\frac{j\omega\mu}{n^2}\frac{\partial}{\partial y}\mathbf{u}_x\mathbf{u}_x \\ &\quad - \left(\frac{\partial}{\partial x}\frac{j\omega\mu}{n^2}\frac{\partial}{\partial x} + j\omega\mu\right)\mathbf{u}_x\mathbf{u}_y \\ &\quad + \left(\frac{\partial}{\partial y}\frac{j\omega\mu}{n^2}\frac{\partial}{\partial y} + j\omega\mu\right)\mathbf{u}_y\mathbf{u}_x \\ &\quad - \frac{\partial}{\partial y}\frac{j\omega\mu}{n^2}\frac{\partial}{\partial x}\mathbf{u}_y\mathbf{u}_y\end{aligned} \quad (120)$$

where $n^2 = \epsilon\mu - \kappa^2\epsilon_0\mu_0$ and where $\overline{\overline{L}}_{21}$ follows from $\overline{\overline{L}}_{12}$ by replacing μ with $-\epsilon$.

Now assume that $\kappa = \delta\kappa$ is small and that we perform a perturbation analysis, as in (57)–(59), to determine the perturbated operators $\overline{\overline{A}} = \overline{\overline{U}} + \delta\overline{\overline{A}}$, $\overline{\overline{D}} = -\overline{\overline{U}} + \delta\overline{\overline{D}}$, $\overline{\overline{B}} = \delta\overline{\overline{B}}$, and $\overline{\overline{C}} = \delta\overline{\overline{C}}$. From the involution requirement follows that $\delta\overline{\overline{A}} = \delta\overline{\overline{D}} = 0$. Taking only first order terms into account it follows from (32) that

$$\overline{\overline{L}}_{12}\delta\overline{\overline{C}} + \delta\overline{\overline{B}}\overline{\overline{L}}_{21} = -2\delta\overline{\overline{L}}_{11} \quad (121)$$

and

$$\overline{\overline{L}}_{12}\delta\overline{\overline{C}}^T + \delta\overline{\overline{B}}^T\overline{\overline{L}}_{21} = -2\delta\overline{\overline{L}}_{11} \quad (122)$$

where "T" indicates the adjoint operator and where we used the fact that the $\overline{\overline{L}}_{ij}$ operators are self-adjoint. This is a similar set of equations as in (57) and (58) but now we have dyadic operators and not matrices. It follows that the anti-symmetric part of the operators $\delta\overline{\overline{B}}$ and $\delta\overline{\overline{C}}$ vanishes and that the symmetric parts (subscript "S") satisfy

$$\overline{\overline{L}}_{12}\delta\overline{\overline{C}}_S + \delta\overline{\overline{B}}_S\overline{\overline{L}}_{21} = -2\delta\overline{\overline{L}}_{11}. \tag{123}$$

One possible way to actually solve these equations is to represent the operators by infinitely dimensional matrices. It is again clear that the (123) does not fully specify the operators $\delta\overline{\overline{B}}$ and $\delta\overline{\overline{C}}$. The degrees of freedom left could be used to make the eigenvalues of both operators equal.

Finally we want to remark that for a lossless chiral waveguide the transformation

$$\begin{aligned} \gamma &\to -\gamma, \\ \mathbf{E}_t &\to \mathbf{E}_t^*, \\ E_z &\to E_z^*, \\ \mathbf{H}_t &\to -\mathbf{H}_t^*, \\ H_z &\to -H_z^*. \end{aligned} \tag{124}$$

between the modal field patterns can be verified from (115)–(118). This transformation however is nonlinear due to the complex conjugates.

Appendix C

In this Appendix, the general solution of the system (53) and (54) is discussed. First define the $2N \times 2N$-matrices $\overline{\overline{x}}$, $\overline{\overline{a}}$, and $\overline{\overline{j}}$ as

$$\overline{\overline{a}} = \begin{pmatrix} \overline{\overline{P}} & \overline{\overline{Z}} \\ \overline{\overline{Y}} & \overline{\overline{Q}} \end{pmatrix} \tag{125}$$

$$\overline{\overline{x}} = \begin{pmatrix} \overline{\overline{A}} & \overline{\overline{B}} \\ \overline{\overline{C}} & \overline{\overline{D}} \end{pmatrix} \tag{126}$$

and

$$\overline{\overline{j}} = \begin{pmatrix} 0 & \overline{\overline{U}} \\ -\overline{\overline{U}} & 0 \end{pmatrix}. \tag{127}$$

The expression (53) can then be written as

$$\overline{\overline{a}}^T\overline{\overline{j}}\,\overline{\overline{x}} - \overline{\overline{j}}\,\overline{\overline{x}}\,\overline{\overline{a}} = 0. \tag{128}$$

The matrix $\overline{\overline{a}}$ can be diagonalized as

$$\overline{\overline{a}} = \overline{\overline{v}}\,\overline{\overline{d}}\,\overline{\overline{v}}^{-1} \tag{129}$$

where $\overline{\overline{v}}$ is the matrix with the eigenvectors and $\overline{\overline{d}}$ a diagonal matrix with the $2N$ eigenvalues of the matrix $\overline{\overline{a}}$. Inserting the expansion (129) in (128) yields after some manipulations

$$\overline{\overline{d}}\,\overline{\overline{z}} - \overline{\overline{z}}\,\overline{\overline{d}} = 0 \tag{130}$$

with

$$\overline{\overline{z}} = \overline{\overline{v}}^T\overline{\overline{j}}\,\overline{\overline{x}}\,\overline{\overline{v}}. \tag{131}$$

From (130) it follows that $\overline{\overline{z}}$ is a diagonal matrix. The elements on the diagonal are not specified by (130). From the involution condition (54), i.e., $\overline{\overline{x}}\,\overline{\overline{x}} = \overline{\overline{u}}$, with $\overline{\overline{u}}$ the $2N \times 2N$ unit matrix, it follows that the eigenvalues of $\overline{\overline{x}}$ should be ± 1. If we express $\overline{\overline{x}}$ as function of $\overline{\overline{z}}$, by inverting (131), then the eigenvalues λ of $\overline{\overline{x}}$ are solution of

$$\det\left[(\overline{\overline{v}}^T)^{-1}\overline{\overline{z}}\,\overline{\overline{v}}^{-1} - \lambda\overline{\overline{j}}\right] = 0. \tag{132}$$

Since the transposition operator does not change the determinant of a matrix it follows that λ is also solution of

$$\det\left[(\overline{\overline{v}}^T)^{-1}\overline{\overline{z}}\,\overline{\overline{v}}^{-1} + \lambda\overline{\overline{j}}\right] = 0. \tag{133}$$

Hence, if λ is an eigenvalue then also $-\lambda$ is an eigenvalue. If we demand that N of the eigenvalues λ are equal to 1 then the other N eigenvalues automatically satisfy the condition that they are equal to -1. This means that the involution condition only poses N extra conditions on the $2N$ diagonal elements of $\overline{\overline{z}}$. This in turn means that the system (53) and (54) has still N degrees of freedom left. It also follows from (131) that $\overline{\overline{j}}\,\overline{\overline{x}}$ is a symmetric matrix which proves that $\overline{\overline{B}}$ and $\overline{\overline{C}}$ are symmetric matrices and that $\overline{\overline{D}} = -\overline{\overline{A}}^T$.

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References

[1] I. V. Lindell, "On the quasi-TEM modes in inhomogeneous multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, no. 8, pp. 812–817, Aug. 1981.

[2] N. Faché, F. Olyslager, and D. De Zutter, *Electromagnetic and Circuit Modeling of Multiconductor Transmission Lines.* Oxford: Clarendon Press, 1993.

[3] F. Olyslager, D. De Zutter, and A. T. de Hoop, "New reciprocal circuit model for lossy waveguide structures based on the orthogonality of the eigenmodes," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 12, pp. 2261–2269, Dec. 1994.

[4] D. De Zutter and F. Olyslager, "High-frequency reciprocity based circuit model for the incidence of electromagnetic waves on general waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 8, pp. 1826–1833, Aug. 1995.

[5] F. Olyslager, "High-frequency reciprocity based circuit model for the incidence of electromagnetic waves on general circuits in layered media," *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 6, pp. 862–873, June 1996.

[6] F. Olyslager and D. De Zutter, "High-frequency transmission line model for a thin wire above a conducting ground," *IEEE Trans. Electromag. Comp.*, vol. 37, no. 2, pp. 234–240, May 1995.

[7] P. R. McIsaac, "Mode orthogonality in reciprocal and nonreciprocal waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 11, pp. 1808–1816, Nov. 1991.

[8] F. Olyslager, E. Laermans, and D. De Zutter, "Rigorous quasi-TEM analysis of multiconductor transmission lines in bi-isotropic media. Part 1: Theoretical analysis for general inhomogeneous media and generalization to bianisotropic media," *IEEE Trans. Microwave Theory and Tech.*, vol. 43, no. 7, pp. 1409–1415, July 1995.

[9] ——, "Rigorous quasi-TEM analysis of multiconductor transmission lines in bi-isotropic media. Part 2: Numerical solution for layered media," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 7, pp. 1416–1423, July 1995.

[10] R. F. Harrington and A. T. Villeneuve, "Reciprocity relationships for gyrotropic media," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 308–310, July 1958.

[11] A. T. Villeneuve, "Orthogonality relationships for waveguides and cavities with inhomogeneous anisotropic media," *IRE Trans. Microwave Theory Tech.*, vol. MTT-7, pp. 441–446, Oct. 1959.

[12] C. Altman and K. Suchy, *Reciprocity, Spatial Mapping and Time Reversal in Electromagnetics.* Norwell, MA: Kluwer, 1991, pp. 62–74.

[13] J. Van Bladel, *Electromagnetic Fields.* Washington: Hemisphere Publishing Corp., 1984, p. 234.

[14] I. V. Lindell, *Methods for Electromagnetic Field Analysis.* Oxford: Clarendon Press, 1992.

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